Assume that we have functions for generating the standard 3D (4 x 4) transformation matrices, \texttt{RotateX(degrees), RotateY(degrees), RotateZ(degrees, Translate(x, y, z)}, and \texttt{Scale(sx, sy, sz)}. You can refer to these by name (and don’t bother to multiply them out).

1. Write down the matrix \texttt{RotateX(θ)} for arbitrary angle θ. Verify by direct calculation that this matrix is always orthonormal. (You can ignore the fourth column \([0 0 0 1]^T\)).

2. The figure below shows a frame (coordinate system) with the standard basis \{e1, e2\} along with a second frame centered at (11, 4). The basis \{v1, v2\} for the new frame is rotated 45 degrees counterclockwise and the y-axis is flipped. You can assume \(v1\) and \(v2\) are orthogonal and have unit length.
a) Derive the transformation matrix that transforms the original frame to the new frame. Indicate how you got it. Use it to calculate the coordinates of the transformed triangle PQR (shaded) with respect to the original frame. Verify geometrically that your results make sense (i.e., round to one decimal place and check that the distances are approximately correct.)
(continued)

b) Derive the change-of-frames matrix that gives you the coordinates of objects with respect to the new frame. Indicate how you got it. Use it to calculate the coordinates of the points P, Q, and R with respect to the new frame. Verify geometrically that your results make sense.

3. Suppose you have an object centered at (1, 2, 3). What is the transformation matrix that will rotate the object by d degrees ccw about a vertical axis through its center? (Assume that the y-axis is vertical.)

4. Suppose you have a sphere-like object centered at (0, 2, 3) whose vertical axis is tilted by 23.4 degrees from the vertical (like the earth). Assume its axis is in the y-z plane (where y is vertical) and is tilted in the positive z-direction. Find the transformation matrix you would use to rotate the object d degrees ccw about its axis.
5. A direction can be described using two angles, an *inclination* $\theta$ from the vertical, also called the *polar angle*, and a rotation $\varphi$ about the vertical, also called the *azimuth*. (Note that this direction corresponds to the location of the $y$-axis after rotations by pitch angle $\theta$ and head angle $\varphi$.)

![Directional Angles](image)

Write a function `Rotate(theta, phi, degrees)` that returns the matrix for a transformation that rotates an object *degrees* degrees ccw about an axis (through the origin) described by the given angles *theta* and *phi*.

Aside: if you also give a radial distance $r$ from the origin, the three values $r$, $\theta$, and $\varphi$ determine a unique point and are called *spherical coordinates* for the point. Note that in calculus books, the $z$-axis is vertical and the azimuth is the $x$-axis, and they typically use symbol $\theta$ for the azimuth and $\varphi$ for the polar angle, the opposite what we’re using here.