Problem-set 2: Higher-order Functions

Problem 1

In order to build a set of elements of type \( a \) we can use a property function \( f \) with \( f :: a \rightarrow \text{Bool} \). We write \( \{ x \mid f(x) \} \), called set builder, to define a set of type \( a \) where all elements \( x \) satisfy property \( f \), i.e. \( \forall x, f(x) = \text{true} \). In fact, the “characterizing” function \( f \) yields true for elements of the set and it yields false for all other arguments (e.g. \( f = \text{isPrime} \), \( f(3) = \text{true} \), \( f(6) = \text{false} \)). Note, a set builder is a function that uses a predicate to build a concrete set.

Implement the module \texttt{SetOperations} that provides the data type \texttt{Set a} – a “functional type”, and the following set operations:

1. \texttt{makeset} :: (\( a \rightarrow \text{Bool} \)) \rightarrow \texttt{Set a}, which takes a property function and yields the corresponding set,
2. \texttt{union} :: \texttt{Set a} \rightarrow \texttt{Set a} \rightarrow \texttt{Set a}, the union of two sets,
3. \texttt{intersect} :: \texttt{Set a} \rightarrow \texttt{Set a} \rightarrow \texttt{Set a}, the intersection of two sets,
4. \texttt{difference} :: \texttt{Set a} \rightarrow \texttt{Set a} \rightarrow \texttt{Set a}, the difference of two sets,
5. \texttt{symdiff} :: \texttt{Set a} \rightarrow \texttt{Set a} \rightarrow \texttt{Set a}, the symmetric difference of two sets, and
6. \texttt{member} :: \( a \rightarrow \texttt{Set a} \rightarrow \texttt{Bool} \), a function to check whether a specific object is element of a given set.

Examples of set builders:

1. \texttt{makeset even}: the set builder for all even integers,
2. \texttt{makeset (\( \lambda a \rightarrow \text{x mod 6 == 0} \))}: the set builder for all integers that are multiples of 6,
3. \texttt{union (makeset even) (makeset (\( \lambda a \rightarrow \text{x mod 6 == 0} \))}): the set builder for all even integers, which are multiples of 6.
Problem 2

Define the function \texttt{complement :: Set a -> Set a} that yields the complement of a given set.

Problem 3

If possible, define the operator \texttt{==} (equality) for the data type \texttt{Set a}. If it is not possible to define the operator \texttt{==}, explain why.

Problem 4

Based on the type \texttt{Set a}, define the data type \texttt{FSet a} that is equipped with the carrier set, and the function \texttt{makefset :: Set a -> [a] -> FSet a}, which yields a finite set of type \texttt{a}. Furthermore, if possible, define the operator \texttt{==} for the data type \texttt{FSet a}.

Submission deadline: Thursday, October 4, 2001, 11am

Submission procedure: Submissions should be made electronically. Send your solution to pramod@cs.iastate.edu with a CC to lumpe@cs.iastate.edu.

On the department’s computer systems (HP’s), use the command \texttt{hugs} to start the Haskell interpreter.