Batting Flying Objects to a Target

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Abstract—This paper presents a planning algorithm for a 2-DOF robotic arm to bat flying 2D objects to a targeted location. Impact dynamics are combined with trajectory kinematics and manipulator dynamics to compute the evolving set of states (poses and motions) of the arm able to achieve the task as the object is flying. Planning is conducted under the arm’s dynamic and kinematic constraints. At the time of hit, the robot executes an action to minimize its total energy. Simulation has been conducted based on the parameter values of a Whole Arm Manipulator (WAM) from Barrett Technology, Inc., with which experiment is undergoing.

I. INTRODUCTION

Impact happens when two or more bodies collide. Its short duration (< 0.1 second) yields a high impulsive force. Kinetic energy is first stored and then released or transferred among the involved bodies. Impulsive forces possess an efficiency edge over static and dynamic forces. On many occasions, we take advantage of impact to accomplish tasks that would otherwise be difficult, if not impossible. Sports, for instance, are an area where impact is used often. Just consider actions like serving a topspin in table tennis, batting a baseball, making a pool shot, etc.

Rather than avoiding a collision, the robot should try to leverage the phenomenon in a manipulation task. Since impact control has to be open-loop due to the short time period, accuracy and robustness could be compromised if impact is not carefully planned.

In this paper, we consider how to bat flying objects so they are redirected to intended targets. For impact planning we need to decide when to strike the object, which spot to strike on, and at what velocity. Such planning needs to be combined with trajectory planning since the object will be in a free motion after the strike.

Relatively little work exists on impact planning, despite some noticeable efforts on impulsive manipulation [7], [14], [3], [4], and [13]. Related to batting, the work [1] focused on the swing trajectory and the force/torque required to generate it, applying Newton’s kinematic restitution law [8]. A high-speed robot system [10] was designed for batting the baseball using a hybrid trajectory. The work was then extended to control the direction of the post-impact ball motion [11], though not the entire trajectory.

Swinging a bat to hit an object is much easier to perform than to analyze in terms of mechanics. We will start with a simple version of the batting problem. A self-actuated bat shown in Fig. 1 is controlled to hit a 2D object in order to alter its trajectory to reach some destination point \( d \) (where, say, a target or container is placed). Three assumptions are made:

1) no contact friction between the bat and the object;
2) known positions and orientations of the bat and the object at the moment of impact;
3) known velocity and angular velocity of the object just before the impact.

Section II presents impact dynamics and energy-based restitution, and then describes how to obtain the bat’s pre-impact motion to drive the object to the destination, when prescribed a hitting configuration. Section III considers a realistic setting of the bat driven by a two-link robotic arm. Combining inverse kinematics, impact dynamics, and motion trajectory, we describe a two-dimensional set of the arm’s joint angles and velocities to achieve the batting task. A pre-impact arm state from the set is selected via minimizing the total energy of the arm and the bat. Section IV describes a batting algorithm that plans the arm’s motion during the object’s flight, and offers several examples. The final section discusses on immediate and future extensions.

Vectors in the paper are by default column vectors. A tuple is written as a row vector. The subscripts \( x \) and \( y \) of a letter (not bolded) represent the \( x \)- and \( y \)-coordinates (or components) of a point (or a vector) named by the same letter (bolded), respectively. For instance, \( p_x \) denotes the \( x \)-coordinate of the point \( p \), while \( V_{ox} \) the \( x \)-component of a velocity \( V_o \). The superscripts ‘−’ and ‘+’ refer to quantities before and after the impact, respectively. The subscript \( \perp \) of a vector rotates the original vector through \( \pi/2 \). For instance, given a vector \( \mathbf{v} = (v_x, v_y) \), \( \mathbf{v} \perp = (-v_y, v_x) \) such that \( \mathbf{v} \times \mathbf{u} = \mathbf{v} \perp \mathbf{u} \) for any vector \( \mathbf{u} \).

All units are from the metric system. In particular, we use second (s) for time, meter (m) for length, radian (rad) for angle, kilogram (kg) for mass, kilogram square meter
(kg \cdot m^2) for moment of inertia, Newton (N) for force, and Joule (J) for work and energy. Units will be omitted from now on.

II. TWO-DIMENSIONAL FRICTIONLESS IMPACT

Fig. 2 shows the impact configuration, where the object was flying towards the left just before the impact. The vectors \( r_o \) and \( r_b \) locate the contact point \( p \) relative to the centers of mass \( o_o \) and \( o_b \) of the object and the bat, respectively. Without loss of generality, the target \( d \) lies to the right of the point of impact \( p \). The contact normal \( \hat{n} = (n_x, n_y)^T \) has \( n_x > 0 \), which is a necessary condition for successful batting.

Let \( m_b \) be the mass of the bat, and \( m_o \) that of the object. Denote by \( V_b \) and \( V_o \) the velocities of the bat and the object, respectively, and by \( \omega_b \) and \( \omega_o \), their angular velocities. The impact changes the object’s velocity instantly to \( V_o^+ = V_o + \Delta V_o \). The object will perform a free flying motion under gravity. Our task here is to plan the bat’s pre-impact velocities \( V_b^+ \) and \( \omega_b^+ \), so the object’s trajectory is altered to pass by the destination \( d \).

A. Impact Dynamics

Let \( I \) be the impulse exerted by the bat on the object. An opposite impulse \(-I\) is exerted on the bat by the object under Newton’s third law. By Newton’s second law, the velocities change during the impact are (refer to Fig. 2 for the configuration)

\[
\begin{align*}
\Delta V_b &= -\frac{1}{m_b}I, \\
\Delta \omega_b &= -\frac{1}{\sigma_b}r_b \times I, \\
\Delta V_o &= \frac{1}{m_o}I, \\
\Delta \omega_o &= \frac{1}{\sigma_o}r_o \times I, \\
\end{align*}
\]

where \( \sigma_b \) and \( \sigma_o \) are the moments of inertia of the bat and object, respectively. Let \( r_i \perp = (r_{iy}, r_{ix})^T, i = b, o \) be the vectors perpendicular to \( r_i \). Let \( v_b \) and \( v_o \) be the velocities of the two points on the bat and the object coinciding at the contact \( p \). We have

\[
\begin{align*}
v_b &= V_b + \omega_b r_b \perp, \\
v_o &= V_o + \omega_o r_o \perp. \\
\end{align*}
\]

Here, \( v_b \) is referred to as the batting velocity. The contact velocity is

\[
v = v_o - v_b = V_o + \omega_o r_o \perp - V_b - \omega_b r_b \perp.
\]

After plugging in (1), it changes by the amount

\[
\Delta v = \Delta v_o - \Delta v_b = \frac{I}{m_o} + \frac{r_o \times I}{\sigma_o} r_o \perp + \frac{1}{m_b}I + \frac{r_b \times I}{\sigma_b} r_b \perp.
\]

where

\[
S = \frac{m_o + m_b}{m_o m_b} \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) + \frac{1}{\sigma_o} r_o \perp r_o \perp + \frac{1}{\sigma_b} r_b \perp r_b \perp.
\]

is positive definite.

B. Energy-based Restitution

Impact is divided into two phases [9, p. 212]: compression and restitution. During compression, the kinetic energy is transformed into the potential energy \( E \) stored at the contact. The phase ends with zero velocity and maximum energy \( E_{\text{max}} \). The elastic portion of the stored energy, of the amount \( e^2 E_{\text{max}} \), is released during the following restitution phase. Here \( e \in [0,1] \) is referred to as the energetic coefficient of restitution. The energy loss by the factor of \( 1-e^2 \) is due to deformation, heat, etc.

Absence of friction means that there is no impulse or energy exchange along the tangent direction. Hence \( I = \hat{n} I \), where \( I \) is the impulse magnitude. It is convenient to describe the impact process in terms of \( I \) instead of time. The energy function \( E \) is differentiable during each impact phase:

\[
\frac{dE}{dI} = -\hat{n}^T v = -\hat{n}^T (v^- + \Delta v) = -v^- \hat{n} + \hat{n}^T S \hat{n} I,
\]

where \( v^- \) is the contact velocity, and \( v^- = \hat{n}^T v^- \) its normal component.

At the end of compression, \( dE/dI = 0 \), which, after substitution of (6), yields the impulse value \( I_c = -v^- / (\hat{n}^T S \hat{n}) \). Integrate (6) from 0 to \( I_c \) to obtain the energy at \( I_c \):

\[
E_{\text{max}} = \frac{(v^-)^2}{2\hat{n}^T S \hat{n}}. 
\]

Restitution begins with the energy \( e^2 E_{\text{max}} \) and decreases it to zero when the phase ends with the impulse value \( I_c \). We have

\[
e^2 E_{\text{max}} = \int_{I_c}^I dE, 
\]

from which we solve for the total impulse:

\[
I_r = \frac{1}{n^T S \hat{n}} (1 + e)(\hat{n}^T v^-) = -\frac{(1 + e)v^-}{n^T S \hat{n}}. 
\]

C. Bat Motion

With the impact outcome derived, we are now ready to plan the motion of the bat. Substitute (9) into (1):

\[
\Delta V_o = \frac{I}{m_o} \hat{n} = \frac{- (1 + e)(v^- - \hat{n} v^-)}{m_o n^T S \hat{n}} \hat{n} = \frac{- (1 + e)(v^- - \hat{n} v^-)}{m_o n^T S \hat{n}} \hat{n},
\]

where \( v^- = \hat{n}^T v^- \), and \( v^- = \hat{n}^T v^- \) are the pre-impact normal components of \( v_o \) and \( v_b \), respectively. Notice that the tangential component of \( v_b \) will not affect the motion of
the object in the absence of friction. From (10), the object’s post-impact velocity is linear in $v_{bn}^-$:

$$V_o^+(v_{bn}^-) = V_o^- + \Delta V_o = V_o^- - \frac{(1 + \epsilon)(v_{on}^- - v_{bn}^-)}{m_o \hat{n}^T \hat{n}} \hat{n}. \quad (11)$$

Let $q = d - p + r_o$. For the object to pass through $d$, the following kinematic equation needs to be satisfied:

$$q = V_o^+ t - (0, 0.5g)^T t^2, \quad (12)$$

for some flight time $t$, where $g$ is the gravitational acceleration. We first obtain $t$ by taking the cross products of both sides of (12) with $(0, 0.5g)^T$. Next, we take their dot products with $(0, 0.5g)^T$, plugging in the obtained expression for $t$. This yields an equation in terms of the $x$- and $y$-coordinates of $q$, $V_o^+$, and $r_o$,

$$q_y = \frac{V_{ox}^+ v_{ox}^- q_x - 0.5 g v_{ox}^-}{V_{ox}^+}. \quad (13)$$

Knowing $q$, we substitute (11) into (13) to obtain a quadratic equation in the normal batting velocity $v_{bn}^-$. Such a velocity exists if and only if the following two conditions are satisfied:

$$\left(\frac{a_x v_{ox}^- q + V_{ox}^- \hat{n} \times q}{2} - 4 a_x q \times \hat{n} \right) (V_{ox}^- q - 0.5 g q_{x}) \geq 0, \quad (14)$$

for some flight time $t$, where $g$ is the gravitational acceleration. We first obtain $t$ by taking the cross products of both sides of (12) with $(0, 0.5g)^T$. Next, we take their dot products with $(0, 0.5g)^T$, plugging in the obtained expression for $t$. This yields an equation in terms of the $x$- and $y$-coordinates of $q$, $V_o^+$, and $r_o$.

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The first condition, independent of the masses $m_o$ and $m_b$, and the restitution coefficient $\epsilon$, ensures that a real root of (13) exists. The second condition, on the found root under the first condition, ensures the impact to happen. Together they are called task conditions. The task conditions guarantee the object to reach the destination $d$.

A detailed analysis (omitted for lack of space) has been performed to characterize the region of $(q_x, q_y)^T$ that admits a solution to equation (13). The region is rather large, which means that the bat is capable of sending the object to a wide range of locations.

### III. IMPACT PLANNING

In a real situation, the bat is attached to a robotic manipulator. This section plans the motion of a two-link arm to carry out the batting task during the flight of an object. We will first work out the arm’s inverse kinematics to attain a specific batting configuration. Then we will describe a two-dimensional set of joint angles and velocities with which batting can send an object, at a fixed moment during its flight, to the target. Next, optimization will be introduced to minimize the total energy of the arm and the bat (which measures the work done by the arm).

#### A. Inverse Kinematics

As shown in Fig. 3, we first determine the pose of a two-link robot arm enabling the bat to hit the flying object at $p$. Suppose that the contact normal $\hat{n}$ is known. The two links attached to the arm joints $J_1$ and $J_2$ have lengths $l_1$ and $l_2$, respectively. Let $l_b$ be the length of the bat. We would like to determine the joint angles $\theta_1$ and $\theta_2$.

The origin is placed at $J_1$. The unit vectors along the two links are

$$\hat{\mathbf{l}}_1 = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix} \quad \text{and} \quad \hat{\mathbf{l}}_2 = \begin{pmatrix} \cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) \end{pmatrix}. \quad (16)$$

Thus, $J_2$ is at the location $l_1 \hat{\mathbf{l}}_1$. Immediately, we set up three conditions:

$$\hat{n}^T \mathbf{l}_2 = 0, \quad (17)$$

$$\hat{n}^T (p - l_1 \hat{\mathbf{l}}_1) = 0, \quad (18)$$

$$\hat{l}_2^T (p - l_1 \hat{\mathbf{l}}_1) \in (l_2, l_2 + l_b). \quad (19)$$

Equation (18) implies that $|\hat{n}^T p/l_1| = |\hat{n}^T \hat{l}_1| \leq 1$. Thus, batting at $p$ can be performed only if the reachability condition below is satisfied:

$$-l_1 \leq \hat{n}^T p \leq l_1. \quad (20)$$

Let $\alpha$ be the polar angle of $\hat{n}$, and $\beta = \cos^{-1}(\hat{n}^T \hat{l}_1)$. See Fig. 3(b). Then we have

$$\theta_1 = \alpha \pm \beta, \quad (21)$$

corresponding to two possible configurations of link 1.

Substitute (16) for $\hat{l}_2$ into (17) and rewrite it as

$$\cos \theta_2 \left( \hat{\mathbf{l}}_1^T \hat{n} \right) + \sin \theta_2 (\hat{\mathbf{l}}_1 \times \hat{n}) = 0. \quad (22)$$

Since the vector $\hat{l}_1^T \hat{n} \hat{l}_1 \times \hat{n}$ is unit, the above yields $\cos \theta_2, \sin \theta_2 = \pm (\hat{n} \times \hat{l}_1, \hat{n}^T \hat{l}_1)$, from which we obtain

$$\theta_2 = \pi \pm \arctan(\hat{n}^T \hat{l}_1, \hat{n} \times \hat{l}_1). \quad (22)$$

In the above, the sign is uniquely chosen under condition (19) because $l_2$ must be in the direction of $\hat{l}_2$. For each $\theta_1$ value, at most one $\theta_2$ value exists.

In summary, conditions (17)–(19) induce at most two possible values of $(\theta_1, \theta_2)$, for which the configurations are drawn in Fig. 3(b).
B. Configuration Space of Contact

For simplicity we assume convexity of the object. At the time of impact, its center of mass \( o_0 \) is at \( p - r_o \) (cf. Fig. 2), while its body frame centered at \( o_0 \) has a rotation from the world frame described by the matrix \( R \). Let the curve \( \gamma(s) = (\gamma_x(s), \gamma_y(s))^T \) describe the object’s boundary in the body frame such that the parameter \( s \) increases counterclockwise. By a slight abuse of notation, we let \( s \) locate the contact point \( p \) on \( \gamma \), i.e., \( R\gamma(s) = r_o \). The contact normal is thus

\[
\hat{n}(s) = R(-\gamma'_x, -\gamma'_y)^T / \|\gamma'\|. \tag{23}
\]

Below we describe the set of \((\theta_1, \theta_2)\) values that satisfy the constraints (17)–(19) as the contact point

\[
p(s) = o_o + R\gamma(s) \tag{24}
\]

varies along the object’s boundary under the conditions (20) and \( n_x > 0 \).

Since \( \hat{n} \) and \( p \) depend on \( s \), so do the angles \( \alpha \) and \( \beta \). Equation (21) defines \( \theta_1 \) as a function of \( s \), where the sign ‘+’ or ‘−’ is chosen over different intervals of \( \theta_1 \). In the joint angle space, as \( s \) varies, all \((\theta_1, \theta_2)\) that result in the bat making contact with the object form curve segments. These segments are further reduced by the joint ranges \( \Theta_1 \) of \( \theta_1 \) and \( \Theta_2 \) of \( \theta_2 \).

C. Configurations for Batting

Consider an arm pose \((\theta_1, \theta_2(\theta_1))\) allowing the bat to be in contact with the object at the current time instant. For convenience, the contact point on the bat is \( l_1 \hat{I}_1 + a \hat{I}_2 \), where \( a = \hat{I}_2^T(p - l_1 \hat{I}_1) \) depends on \( \theta_1 \) and \( \theta_2 \). The velocity of the contact point just before the impact is obtained through differentiation while treating \( a \) as a constant:

\[
v_b^- = l_1 \hat{I}_1 \dot{\theta}_1 + a \hat{I}_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2), \tag{25}
\]

where the subscript ‘\( \perp \)’, introduced in the end of Section I, rotates the original vector through \( \pi / 2 \). In the rest of the paper, we will use \( \dot{\theta}_1 \) and \( \dot{\theta}_2 \), without the superscript ‘\( \perp \)’, to denote the joint velocities just before the impact.

For \( k = 1, 2 \) let \( m_k \) be the mass of link \( k \), \( \sigma_k \) its moment of inertia about the joint \( J_k \), and \( d_k \) the distance from its center of mass to the joint \( J_k \). Let \( d_0 \) be the distance from the bat’s center of mass to the joint \( J_2 \). Then the bat’s moment of inertia about \( J_2 \) is \( \sigma_b = \sigma_b + m_b d_0^2 \).

The bat exerts an impulsive force on the flying object, as well as on link 2 to which it is rigidly attached. Link 1 is not engaged in the impact due to the joint \( J_2 \) and the very short impact duration. Since the joint angle \( \theta_1 \) is not changing, we have from (25) during the impact

\[
\Delta v_b = \hat{a} \hat{I}_2 \Delta \dot{\theta}_1, \tag{26}
\]

where the change in the angular velocity of link 2 about \( J_2 \) is, from dynamics,

\[
\Delta \dot{\theta}_2 = -\frac{\hat{a} \times \hat{I}_2}{\sigma_2 + \sigma_b} \tag{27}
\]

The velocity \( v_b \) of the contact point on the object is still given in (3). The change in the contact velocity \( \Delta v \) assumes the same form (4) except

\[
S = \frac{1}{m_o} \left[ \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] + \frac{1}{\sigma_o} r_o \hat{I}_2^T + \frac{a^2}{\sigma_2 + \sigma_b} \hat{I}_2 \hat{I}_2^T. \tag{28}
\]

The analytic results from Sections II-B and II-C subsequently carry over.

For the pre-impact normal velocity \( v_{b1}^- = \hat{n}^T v_b^- \) to exist, the task conditions (14) and (15) must be satisfied by the normal \( \hat{n} \) and \( q = d - p + r_o \). After substitutions of (23) and (24), the conditions are on the parameter \( s \) locating the contact point. They are often satisfied on one boundary segment of the object. Meanwhile, every \((\theta_1, \theta_2)\) satisfying conditions (17)–(19) and lying in the joint angle ranges determines an \( s \) value. Namely, \( s \) is a function of \( \theta_1 \). The tasks conditions are thus imposed on \( \theta_1 \), though their forms in terms of \( \theta_1 \) cannot be written out compactly.\(^1\)

The task conditions (14) and (15) further reduce those \((\theta_1, \theta_2)\) segments from Section III-B which already ensure the bat’s contact with the object and no exceeding of the joint ranges \( \Theta_1 \) and \( \Theta_2 \). Every pair \((\theta_1, \theta_2)\) on a resulting segment is called a feasible pose.

The arm will be based on a 4-DOF WAM arm, where \( \theta_1 \) and \( \theta_2 \) refer to the angles of its joints 2 and 4, and the other two joints are not used. Table I gives the values of physical parameters related to the two links, the bat, and the object.

\[
\begin{align*}
\theta_1 & = 0.55, \quad d_1 = 0.1446, \quad m_1 = 5.6772, \quad \sigma_1 = 0.2931, \\
\theta_2 & = 0.35, \quad d_2 = 0.3426, \quad m_2 = 1.0651, \quad \sigma_2 = 0.04076, \\
\theta_b & = 0.265, \quad d_b = 0.44, \quad m_b = 0.3433, \quad \theta_b = 0.0745, \\
\Theta_1 & = [-0.429, 3.571], \quad \Theta_1 = [-0.85, 0.85], \quad \Phi_1 = [-8, 8], \\
\Theta_2 & = [\pm 0.9, \pm 3.1], \quad \Theta_2 = [-5, 5], \quad \Phi_2 = [-60, 60].
\end{align*}
\]

**TABLE I:** Values of physical parameters. The first four rows display the lengths, masses, and moments of inertia of the two links, the bat, and the elliptic object in Figs. 4. The last two rows display the ranges \( \Theta_1, \Theta_2 \), and \( \Phi_1, \Phi_2 \), of joint \( i \)’s angle, velocity, and acceleration. The joint angle ranges are relative to the zero position with arms. The range \( \Theta_2 \) is either \([-0.9, 3.1]\) or \([-3.1, 0.9]\) by controlling the WAM’s third joint.

Fig. 4 plots the area swept out by segments of feasible poses \((\theta_1, \theta_2)\) of the WAM Arm during the flight of a solid elliptic object. At time 0.29, all the feasible poses form the golden segment in the middle of Fig. 4(a). The cross at \((0.4333, 0.8812)\) on the segment defines the batting configuration right below. The contact points, in a one-to-one correspondence to these feasible poses, form the gold section on the ellipse in its current pose as plotted in Fig. 4(b).

At time 0.45, the feasible poses constitute two separate segments, displayed at the top and bottom in part (a). Their purple sections are formed by pairs of arm poses in which the bat contacts the same point on the ellipse. One such pair, comprising \((1.7410, -2.6227)\) and \((-0.3628, 2.6227)\),

\(^1\)In computation it is easier to first find the \( s \) values satisfying \( u(s) \geq 0 \), with each \( s \) value corresponding to up to two \( \theta_1 \) values.
is marked by the rightmost and leftmost crosses, with their defined batting configurations shown right above or below. The contact points for all the pairs constitute the purple section of the ellipse’s boundary shown in part (c) of the figure. The red (blue, respectively) section can only be contacted via a pose from the red (blue, respectively) section of the bottom (top, respectively) segment in (a).

D. Feasible Arm Motions

It is time to look at how to find the joint velocities \( \dot{\theta}_1 \) and \( \dot{\theta}_2 \) for the batting. Given a feasible pose \((\theta_1, \theta_2(\theta_1))\), the pre-impact normal velocity \( v_{bn} \) exists to complete the batting task. It is solved from the quadratic equation resulting from a substitution of (11) into (13). Up to two roots exist. For each value of \( v_{bn} \), it follows from equation (25) that

\[
\left(\mathbf{n}^T \left( l_{11} \mathbf{l}_{1\perp} + a l_{2\perp} \right) \right) \dot{\theta}_1 + a \left(\mathbf{n}^T l_{2\perp} \right) \dot{\theta}_2 = v_{bn}.
\]

Since \( \mathbf{n} \), \( \mathbf{l}_{1\perp} \), and \( \mathbf{l}_{2\perp} \) all depend on \( \theta_1 \), the above equation defines \( \dot{\theta}_2 \) as a function of \( \theta_1 \) and \( \dot{\theta}_1 \):

\[
\dot{\theta}_2(\theta_1, \dot{\theta}_1) = \frac{v_{bn} - \mathbf{n}^T l_{1\perp} l_{1\perp} + a l_{2\perp} \dot{\theta}_1}{a \mathbf{n}^T l_{2\perp}}.
\]

The two variables \( \theta_1 \) and \( \dot{\theta}_1 \) completely characterize the arm’s state (configuration and velocity).

Next, we determine the range of \( \dot{\theta}_1 \) given \( \theta_1 \). Suppose that the robot starts moving at time \( \tau \) ahead of when the object reaches the impact configuration. We let every joint \( k, k = 1, 2 \), accelerate at a constant rate during some initial time period \( \tau_k \) to reach \( \dot{\theta}_k \), and then maintain the velocity for the remaining period \( (\tau - \tau_k) \) until the joint angle reaches \( \theta_k \). Under this acceleration scheme, the following two conditions hold:

\[
\dot{\theta}_k \cdot \theta_k > 0, \quad k = 1, 2.
\]  

Both conditions above are imposed on \( \theta_1 \) and \( \dot{\theta}_1 \). It is easy to derive that

\[
\dot{\theta}_k \left( \frac{\tau - \tau_k}{2} \right) = \theta_k, \quad k = 1, 2,
\]  

which, under (27) and (28), ensure, \( \tau > \tau_1, \tau_2 \). Subtractions of the two equations in (28) yields

\[
\tau_2 - \tau_1 = 2 \left( \frac{\theta_1}{\theta_1} - \frac{\theta_2}{\theta_2} \right).
\]

Let \( \delta_1, \delta_2 > 0 \) be the maximum accelerations\(^3\) for joints 1 and 2. Then the following conditions must be satisfied by \( \tau_1 \) and \( \tau_2 \) to be reachable within the time periods \( \tau_1 \) and \( \tau_2 \), respectively,

\[
\tau_k \delta_k \geq |\dot{\theta}_k|, \quad k = 1, 2.
\]

Additionally, enough time must be provided for the acceleration:

\[
\tau = \frac{\tau_1}{2} + \frac{\tau_2}{\dot{\theta}_k} \leq \tau_{\text{max}}, \quad k = 1, 2.
\]

Here, \( \tau_{\text{max}} \) is the period from the time when a reliable velocity estimate is obtained until the (current) time of impact. The arm could not move before a reliable estimate was obtained.

The inequality (30) yields a lower bound for \( \tau_k \), while the equality (31) yields an upper bound. A pair \((\theta_1, \dot{\theta}_1)\) is feasible only if the upper bound is no less than the lower bound. This leads to the following two conditions on \( \theta_1 \) and \( \dot{\theta}_1 \):

\[
\left| \frac{\dot{\theta}_1}{\delta_k} \right| \leq 2 \left( \tau_{\text{max}} - \frac{\theta_1}{\theta_1} \right), \quad k = 1, 2.
\]

Given \( \theta_1 \) (and thus \( \dot{\theta}_2 \) determined), we apply the conditions (27) and (32), together with the ranges \( \Omega_1 \) and \( \Omega_2 \) for \( \dot{\theta}_1 \) and \( \dot{\theta}_2 \) (the latter given in (26)), to obtain an interval \([\eta_1(\theta_1), \eta_2(\theta_1)]\). The interval includes all values of \( \dot{\theta}_1 \) such that the arm’s state determined by \((\theta_1, \dot{\theta}_1)\) will ensure the object to be batted to the target.

A state \((\theta_1, \dot{\theta}_1)\) is feasible if the arm in the state will bat the flying object to the target. Given a feasible \((\theta_1, \dot{\theta}_1)\), the acceleration period \( \tau_1 \) is set to satisfy (31), and \( \tau_2 \) is determined from (29) after calculating \( \dot{\theta}_2 \) and \( \dot{\theta}_2 \).

The region of feasible states is generally two-dimensional. It can become rather complex, partly because the functions \( \dot{\theta}_2(\theta_1) \) and \( \dot{\theta}_2(\theta_1, \dot{\theta}_1) \) do not have clean closed forms.

\(^3\)Namely, the accelerate range is \( \Phi_i = [-\delta, \delta], i = 1, 2 \).

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\(^2\)The WAM Arm achieves these two poses under different joint 3 values.
E. Batting with Minimum Effort

Our next step is to minimize the batting effort, which is characterized as the total mechanical energy of the bat and the two-link arm. The kinetic energies of the two links and the bat can be derived as follows:

\[
T_1 = \frac{1}{2} \sigma_1 \dot{\theta}_1^2,
\]

\[
T_2 = \frac{1}{2} \sigma_2 \dot{\theta}_2^2 + \frac{1}{2} m_d \ddot{\theta}_2^2 + m_d l_1 \cos \theta_2 \left( \dot{\theta}_1^2 + \dot{\theta}_2 \right),
\]

\[
T_b = \frac{1}{2} \dot{\theta}_b^2 + \frac{1}{2} m_b \ddot{\theta}_b^2 + m_b l_1 \cos \theta_2 \left( \dot{\theta}_1^2 + \dot{\theta}_2 \right).\]

Meanwhile, their potential energies are

\[
U_1 = m_1 g d_1 \sin \theta_1,
\]

\[
U_2 = m_2 g \left( l_1 \sin \theta_1 + d_2 \sin (\theta_1 + \theta_2) \right),
\]

\[
U_b = m_2 g \left( l_1 \sin \theta_1 + d_2 \sin (\theta_1 + \theta_2) \right).
\]

The total energy of the arm is thus

\[
E(\theta_1, \dot{\theta}_1) = T_1 + T_2 + T_b + U_1 + U_2 + U_b.
\]

Minimization of \( E \) is carried out in two steps: first over \( \dot{\theta}_1 \) given \( \theta_1 \), and then over all discretized \( \theta_1 \) values. Discretize the subdomain of object’s boundary curve \( \gamma(s) \) with inward normal \( n \) satisfying \( n_x > 0 \). Let \( s_1, s_2, \ldots, s_n \) be the discretized values that also satisfy the reachability (20), and the task conditions (14) and (15). For every \( s_k \), we substitute in (24) for \( p \), and solve for up to two \( (\theta_1(s_k), \theta_2(s_k)) \) values that satisfy conditions (17)–(19) and fall within the arm’s joint angle range. For each feasible \( (\theta_1(s_k), \theta_2(s_k)) \), use the method in Section III-D to obtain the interval \([n_0, n_0(\theta_1)]\) of feasible joint velocity values \( \dot{\theta}_1 \).

For simplicity, rewrite (26) as \( \dot{\theta}_2 = \lambda_1 + \lambda_2 \dot{\theta}_1 \) where

\[
\lambda_1(\theta_1) = \frac{v_{bn}}{a n^T \hat{L}_{2⊥}} \text{ and } \lambda_2(\theta_1) = -1 - \frac{v_{bn}^T \hat{L}_{2⊥}}{a n^T \hat{L}_{2⊥}}.
\]

The partial derivative \( \partial E / \partial \dot{\theta}_1 \) vanishes at

\[
\eta_c(\theta_1) = -\left( (\sigma_2 + \sigma_0) \lambda_1 + (m_d^2 + m_b^2) l_1 \cos \theta_2 \right)
\]

\[
\Bigg/ \left( \sigma_1 + (\sigma_2 + \sigma_0) l_2 + 2(m_d + m_b) l_1^2 \right)
\]

\[
+ 2(m_d^2 + m_b^2) l_1 \cos \theta_2 (1 + \lambda_2) \Bigg).\]

Then the minimum of \( E \) over \([\eta_0, \eta(\theta_1)]\) for a given \( \theta_1 \) must be achieved at either of the two endpoints, or at \( \eta_c(\theta_1) \) if \( \eta_c(\theta_1) \in [\eta_0, \eta(\theta_1)] \). Denote by \( \xi(\theta_1) \) the value of \( \dot{\theta}_1 \) at which the minimum is achieved.

The optimal pose and motion \( (\theta_1^*, \xi(\theta_1^*)) \) then minimizes \( E \) over all \( (\theta_1, \xi(\theta_1)) \) with feasible pose \( (\theta_1, \theta_2(\theta_1)) \) and non-empty interval \([\eta_0, \eta(\theta_1)]\).

IV. Batting Algorithm and Simulation

Algorithm 1 combines the components from Section III to control the arm to execute a batting operation. The algorithm assumes a Kalman filter (KF) to constantly estimate the motion of the flying object. The filter, being developed and under tuning, uses the image of a “T” marked on the object’s surface as observables. Planning starts immediately when the estimate of the object’s motion converges enough (line 3). At this moment, it hypothesizes the hit to happen at time \( \tau \) ahead (line 5) and check for feasible states of the arm (lines 7–9). If the set of feasible states is big enough, it computes the optimal arm movement and executes the batting operation (lines 11–15). In the case of no good set of feasible states found, it looks for a later hitting time, and repeats so until the object would be out of reach (lines 17–18). A new iteration of the while loop of lines 3–19 begins at the next time instant of the object’s flight.

Consider a scenario of the same elliptic object from Fig. 4. At time 0, the ellipse was at \((1, 5, 7)^T\) with velocity \((-3, 1.5)^T\) and angular velocity 10. Impact happens at time 0.43 when the object reaches \((0.2100, 0.4390)^T\) and completes a rotation of 4.3.

Fig. 5(a) plots the region of feasible states for the WAM Arm in a scenario (shown in (b)) of batting the same elliptic object from Fig. 4. The condition (32) with \( k = 2 \) is easily satisfied with the equality curve lying outside the displayed region. The range \((-0.1952, 0.3190)\) of \( \theta_1 \) has been reduced from \((-0.1952, 2.4383)\) of feasible poses after removing those values that yield no feasible \( \dot{\theta}_1 \).

The coefficient of restitution \( e = 0.8 \) is set for the batting. The optimal state (the blue point) and a non-optimal state (the red point) are selected from the feasible region in (a) for comparison. Their batting outcomes are shown in (b). The post-impact trajectory traced out by solid blue ellipses
is generated by the optimal (blue) state of the arm, also drawn in solid lines, which expands to \( x = (\theta_1, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_2) = (-0.1486, 2.4042, -0.4075, -1.9236) \). Batting expends energy 1.4739, and the object reaches the target at time 1.1031.

The trajectory illustrated by dashed (red) ellipses is the outcome of batting by the arm in the (red) state in Fig. 5(a). The states expands to \( x = (0.0814, 2.3395, 0.5200, -2.0831) \). Not only does this batting expend a much higher energy 4.2877, but also the object reaches the target at a later time 1.248.

Fig. 6 plots the batting of an incoming hexagon. We use the same coefficient of restitution value 0.8. The hexagon has the same mass 0.0175 as the ellipse and the moment of inertia 0.00071542 with respect to the center of mass, where its body frame is located and assumes the same orientation at time 0 as the world frame. At time 0, the hexagon’s center of mass is located at \((1.9, 0.8)^T\). Impact happens at time 0.45 and the contact location \((0.4798, 0.4643)^T\) when the object reaches \((0.5500, 0.4828)^T\) with velocity \((-3.0, -2.91)^T\) and angular velocity 10. The arm’s expanded state is \( x = (-0.1068, 1.8051, -0.3432, -2.2895) \). The post-impact velocities are \((4.9238, -1.8941)^T\) and \(-1.8276\). The hexagon reaches the target at time 0.744.

V. ONGOING AND FUTURE WORK

The planning algorithm (sans vision-based KF estimation) has been tested with the WAM arm under its joint angle, velocity, and acceleration constraints. As of writing, we are in the middle of assembling the planning module with the vision/estimation module to fully automate the system. The work is expected to take another four to eight weeks.

One improvement on Algorithm 1 will be to adjust the planned arm motion, determined according to the motion estimate at a priori time, as more accurate motion estimates are being obtained.

We will extend planning to consider contact friction. The set of feasible arm poses at a time during the object’s flight will have its dimension increased from one to two. The dimensions of feasible states will increase to three accordingly, rendering optimization a more challenging task.

Another extension will be to 3D batting. Technical challenges will include estimation of linear and angular motions of the flying object, and sophisticated planning algorithm design. Tangential compliance [5] may not be ignored as they may play a significant role affecting the impact outcome.

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4Its six vertices in the body frame are \((-0.0672, -0.0247), (-0.0472, 0.0353), (0.0028, 0.0553), (0.0628, -0.0047), (0.0328, -0.0647), (-0.0072, -0.0247)\).

5In a situation like serving a topspin in table tennis.
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