

Planning Finger Movements to Lift up Deformable 2D Objects

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Abstract—This paper describes a strategy to lift up a deformable 2D object resting on a horizontal plane. Inspired by human hand lifting behavior, the strategy plans grasping trajectories of two fingertips using modified Rapidly-exploring Random Trees (RRT). Compared to a straight squeeze, a planned finger movement not only enlarges the graspable region, but also minimizes the work. Improvements on control complexity and robustness are made through modifications to RRT. The introduced strategy is applicable to both hollow and solid 2D objects, and is extendable to 3D ones.

I. INTRODUCTION

It is usually an easy task for a human hand to pick up a deformable object sitting on a table. Placed at certain locations on the object, the fingertips perform coordinated movements until the object is off the table. They usually squeeze the object first, sometimes even downward, to form a firm grip, and gradually turn upward, guided by the brain that is analyzing visual and tactile information, and also based on past experience. However, this wealth of sensor data, or grasping skills, are not possessed by robots.

The special case of straight squeeze grasp was studied in our previous work, with [8] or without [6] considering gravity. Simple as it is, a straight squeeze has some drawbacks. Firstly, it may fail where a curved trajectory succeeds. Secondly, no optimality was considered, i.e. the work done may be more than necessary.

Inspired by human hand grasping, we introduce a strategy to plan the movements of two fingertips to grasp deformable objects. Given an object sitting on a table, and two fingers placed on it, the strategy first calculates the most energy efficient finger movement that grasps and lifts the object, and then generates a path to it subject to physical constraints. It is harder for a robotic finger to follow an arbitrary curve, than a polyline, or a polyline one with more turns than one with less. Here we plan piecewise straight paths with least turns.

For hollow 2D objects, we assume point finger contacts. The displacements of the two fingers can be described together as a point, or a *state*, in a four dimensional configuration space (C-space), bounded by a set of quartic polynomial inequalities with four variables

constructed over physical constraints. The contact force, local contact geometry, and the work can be expressed by the state explicitly. The goal state, in which the object is lifted by minimum work, is then obtained. Thus, the grasping problem reduces to a typical path planning problem with a starting state, a goal state and the C-space. There is a rich literature on path planning [2]. Particularly we use the RRT [7] algorithm to solve the planning problem. We reduce the number of turns on paths planned by RRT to reduce complexity.

We will extend the above strategy to solid objects. Area contact and the changing contact configuration [6] introduce extra difficulty. For instance, no explicit relationship exists between the contact force and the state, which has to be resolved by approximating the contact configuration.

We consider an object to be grasped if in its current shape a rigid body would be under force closure grasp [10]. Extensive work exists on rigid body grasping [1], while much less does for deformable objects [11] [5].

In the rest of the paper, Section II introduces the background on deformable computing, Section III describes the grasping strategy on hollow objects, Section IV extends the strategy to solid objects, Section V shows some simulation results, and discussions follow in Section VI.

II. DEFORMATION UNDER CONTACT AND GRAVITY

A 2D deformable object can be seen as a generalized cylinder swept out by translating a region in the xy -plane along the z -axis by some distance. Its strain energy is $U = \frac{hE}{4(1+\nu)} \iint_S (\frac{2}{1-\nu}(\epsilon_x^2 + 2\nu\epsilon_x\epsilon_y + \epsilon_y^2) + \gamma_{xy}^2) dx dy$, where E is the object's Young's Modulus, h is the thickness, S is the area of the region, ϵ_x and ϵ_y are *elongation*, in the x - and y -direction respectively, and γ_{xy} is *shearing*. Particularly, a ring-like object is obtained if the region is a closed curve $\gamma(s)$, where s is its curve length. Its strain energy is $U = \frac{Ewh}{2} \int_0^L (\epsilon^2 + \frac{h^2}{12}\zeta^2) ds$, where w is the width, L is the length of the center curve, ϵ is the elongation and ζ is the *change in curvature*.

When external force $\mathbf{f}(x, y)$ causes deformation $\delta(x, y)$ on the object, the load potential is $W =$

$-\int \delta^T \mathbf{f}$. Minimization of the potential energy $\Pi = U + W$ yields the deformation at equilibrium.

Generally, without a close form solution, it is solved by the Finite Element Method [3]. The object is represented either as a triangular mesh (solid object), or a ring of line segments (hollow objects) with n vertices $\mathbf{p}_1, \dots, \mathbf{p}_n$, where $\mathbf{p}_i = (x_i, y_i)^T$, for $1 \leq i \leq n$. When the object deforms, \mathbf{p}_i is displaced to $\tilde{\mathbf{p}} = \mathbf{p}_i + \delta_i$. The displacement of any point inside an element is interpolated over those of its vertices. The deformed shape is uniquely described by the *displacement vector* $\Delta = (\delta_1^T, \dots, \delta_n^T)^T$. Then $U = \frac{1}{2} \Delta^T K \Delta$, where K is the symmetric, positive semi-definite stiffness matrix of the object, with three null vectors representing the rigid body motion of translation in the x -, y -direction, and rotation. Let $\mathbf{F} = (\mathbf{f}_1^T, \dots, \mathbf{f}_n^T)^T$ be the force vector, and $\mathbf{G} = (\mathbf{g}_1^T, \dots, \mathbf{g}_n^T)^T$ be the gravity force, the total potential energy is $\Pi = \frac{1}{2} \Delta^T K \Delta - \Delta^T (\mathbf{F} + \mathbf{G})$. Minimizing Π gives the equilibrium equation: $K \Delta = \mathbf{F} + \mathbf{G}$. We can derive [8]

$$\Delta = \sum_{i=1}^{2n-3} \frac{1}{\lambda_i} (\bar{\mathbf{v}}_i^T \bar{\mathbf{F}}) \mathbf{v}_i + (\mathbf{v}_{2n-2}, \mathbf{v}_{2n-1}, \mathbf{v}_{2n}) \mathbf{b} + \mathbf{g} \quad (1)$$

$$0 = \mathbf{v}_i^T (\mathbf{F} + \mathbf{G}), \quad i = 2n-2, 2n-1, 2n \quad (2)$$

where \mathbf{v}_i is the i -th eigenvector of K , \mathbf{b} gathers the projections of Δ onto the null vectors $\mathbf{v}_{2n-2}, \mathbf{v}_{2n-1}, \mathbf{v}_{2n}$, $\mathbf{g} = \sum_{i=1}^{2n-3} \frac{1}{\lambda_i} (\mathbf{v}_i^T \mathbf{G}) \mathbf{v}_i$ is a constant vector. The vector $\bar{\mathbf{x}}$ is composed of the elements from \mathbf{x} that are corresponding to contact nodes. From equation (1) and (2),

$$\begin{pmatrix} \bar{\mathbf{F}} \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} C \\ E^T \end{pmatrix} \bar{\Delta} - M \begin{pmatrix} \bar{\mathbf{g}} \\ (\mathbf{v}_{2n-2}, \mathbf{v}_{2n-1}, \mathbf{v}_{2n})^T \mathbf{G} \end{pmatrix}, \quad (3)$$

where M is a symmetric matrix obtained from the constant coefficients of the equations and is fully ranked if at least three terms of Δ are specified [6]. The reduced stiffness matrix C is $m \times m$, where $m = |\bar{\mathbf{F}}|$, and E is $m \times 3$. And $M = \begin{pmatrix} C & E \\ E^T & H \end{pmatrix}$, where H is 3×3 . With $\bar{\mathbf{F}}$ and \mathbf{b} known, we can solve for Δ from equation (1).

III. GRASP PLANNING FOR HOLLOW OBJECTS

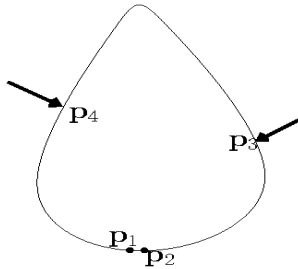


Fig. 1: Grasping a hollow object.

Fig. 1 shows a hollow object sitting vertically on a smooth plane with two contact points \mathbf{p}_1 and \mathbf{p}_2 . Thus at $\mathbf{p}_i, i = 1, 2$, the x -directional supporting force $f_{ix} = 0$. That the plane is horizontal implies $p_{1y} = p_{2y}$ and

$p_{1x} \neq p_{2x}$. Since the two points stay on the table, $\delta_{iy} = 0$.

Initially under gravity \mathbf{G} , each node \mathbf{p}_i displaces by δ'_i . Let $\mathbf{f}'_i = (0, f'_i)^T, i = 1, 2$, be the supporting force at \mathbf{p}_i . Two robotic fingers then make frictional point contact with the object at \mathbf{p}_3 and \mathbf{p}_4 respectively. Then,

$$(F_c^T, F_{sy}^T)^T = C(\Psi^T, \Delta_{sy}^T)^T + (0, f'_1, f'_2)^T, \quad (4)$$

where $F_c = (f_{3x}, f_{3y}, f_{4x}, f_{4y})^T$ is the force vector of finger contacts and $F_{sy} = (f_{1y}, f_{2y})^T$ is the y -directional force vector of supporting points. A state $\Psi = (\psi_{3x}, \psi_{3y}, \psi_{4x}, \psi_{4y})^T$ is the finger displacement. No slip between object and fingers is assumed, so $\Delta_c = (\delta_3^T, \delta_4^T)^T = \Psi + (\delta_3'^T, \delta_4'^T)^T$.

The object is lifted up at the moment $F_{sy} = 0$. We are interested in finding a sequence of Ψ_0, \dots, Ψ_l , such that $\Psi_0 = 0$, and $F_{sy}(\Psi_l) = 0$, subject to some constraints.

A. Reduced Stiffness Matrix

In this subsection, we set $\mathbf{G} = 0$ to analyze some properties of C , since they are independent. Then $\mathbf{f}'_i = \delta'_i = 0$, for $i = 1, \dots, n$, and $\psi_i = \delta_i$ for $i = 3, 4$,

The object's strain energy is now

$$E = (\Psi^T, \Delta_{sy}^T) C (\Psi^T, \Delta_{sy}^T)^T / 2. \quad (5)$$

Lemma 1: The matrix C is symmetric and positive semidefinite, with rank 3.

The proof of this lemma is omitted.

Write $C = \begin{pmatrix} C_1, C_2^T \\ C_2, C_3 \end{pmatrix}$, where C_1, C_2, C_3 are $4 \times 4, 2 \times 4$, and 2×2 submatrices of C . Thus C_1 and C_3 are also symmetric.

Lemma 2: C_1 is positive semi-definite with rank 3.

A proof can be found in Appendix I.

Theorem 3: $\text{Rank}(C_2) = 2$.

Proof: Given C_2 is a 2×4 matrix, $\text{rank}(C_2) \leq 2$. It is obvious that $\text{rank}(C_2) > 0$, because otherwise $\forall \Psi \in \mathbb{R}^4$, when $\Delta_{sy} = 0, F_s = C_2 \Psi = 0$, which immediately violates equilibrium condition and is impossible.

Suppose for contradiction that $\text{rank}(C_2) = 1$. Then $C_2 = (a, b)^T \mathbf{v}^T$, where $a^2 + b^2 \neq 0, \mathbf{v} \in \mathbb{R}^4, \mathbf{v} \neq 0$. Set $\Delta_{sy} = 0$. According to equation (4)

$$(F_c^T, F_{sy}^T)^T = (C_1, C_2^T)^T \Psi = (\Psi C_1, ac, bc)^T \quad (6)$$

where $c = \mathbf{v}^T \Psi$. Denote $\mathbf{f}_3 = (f_{3x}, f_{3y})^T$ and $\mathbf{f}_4 = (f_{4x}, f_{4y})^T$, so $F_c = (\mathbf{f}_3^T, \mathbf{f}_4^T)^T$. Since only the 4 points in contact take non-zero force, the object must be in equilibrium, i.e. $\sum_{i=3}^4 f_{ix} = 0, \sum_{i=1}^4 f_{iy} = 0$, and $\sum_{i=1}^4 \mathbf{p}_i \times \mathbf{f}_i = 0$. Let $d = f_{3x}$, then

$$f_{3x} = -f_{4x} = d. \quad (7)$$

Rewrite the torque equilibrium equation as

$$p_{3x} f_{3y} + p_{4x} f_{4y} = d(p_{3y} - p_{4y}) - (ap_{1x} + bp_{2x})c. \quad (8)$$

Write the y -directional force equilibrium equation and (8) in matrix form

$$\begin{pmatrix} 1 & 1 \\ p_{3x} & p_{4x} \end{pmatrix} \begin{pmatrix} f_{3y} \\ f_{4y} \end{pmatrix} = \begin{pmatrix} 0 \\ p_{3y} - p_{4y} \end{pmatrix} d - \begin{pmatrix} a + b \\ ap_{1x} + bp_{2x} \end{pmatrix} c. \quad (9)$$

In the case $p_{3x} \neq p_{4x}$, the coefficient matrix on the left side is fully ranked, so

$$\begin{pmatrix} f_{3y} \\ f_{4y} \end{pmatrix} = \frac{c}{p_{4x} - p_{3x}} \begin{pmatrix} -(a+b)p_{4x} + (ap_{1x} + bp_{2x}) \\ (a+b)p_{3x} - (ap_{1x} + bp_{2x}) \end{pmatrix} + \frac{d(p_{4y} - p_{3y})}{p_{4x} - p_{3x}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \quad (10)$$

Combine (7) and (10), we have

$$F_c = \frac{c}{p_{4x} - p_{3x}} \begin{pmatrix} 0 \\ -(a+b)p_{4x} - (ap_{1x} + bp_{2x}) \\ 0 \\ (a+b)p_{3x} + (ap_{1x} + bp_{2x}) \end{pmatrix} + \frac{d}{p_{4x} - p_{3x}} \begin{pmatrix} p_{4x} - p_{3x} \\ p_{4y} - p_{3y} \\ p_{3x} - p_{1x} \\ p_{3y} - p_{4y} \end{pmatrix}. \quad (11)$$

In the case $p_{3x} = p_{4x}$, the first row of the right hand side of (9) times p_{3x} is equal to its second row:

$$(ap_{1x} + bp_{2x} - (a+b)p_{3x})c = (p_{4y} - p_{3y})d. \quad (12)$$

Combining equation (7),(12) and (10),

$$F_c = \left(o \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ a+b \\ 0 \\ 0 \end{pmatrix} \right) c + \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} e, \quad (13)$$

for some $e \in \mathbb{R}$, where

$$o = \frac{ap_{1x} + bp_{2x} - (a+b)p_{3x}}{p_{3y} - p_{4y}}$$

is a constant. Note $p_{3y} - p_{4y} \neq 0$ because $p_{3x} = p_{4x}$.

In both equations (11) and (13), all terms on the right hand side, other than c , d and e , depend only on C and contact locations, i.e. they are constant to Ψ . This means F_c is always linearly spanned by at most 2 vectors, which implies that $\text{rank}(C_1) = 2$ given that Ψ is arbitrary. It is a contradiction to Lemma 2. So $\text{rank}(C_2) = 2$. ■

B. Physical Constraints

The object must be fully constrained by the fingers (with supporting plane). So the contact forces at \mathbf{f}_3 and \mathbf{f}_4 always stay inside the friction cone:

$$\mathbf{f}_i \cdot \mathbf{N}_i / |\mathbf{f}_i| \geq 1 / \sqrt{1 + (\kappa\mu)^2}, \quad (14)$$

where $i = 3, 4$, μ is the friction coefficient and $\kappa \in (0, 1]$ is introduced as the *safety parameter* to ensure a

certain margin between the force and the friction cone to tolerate control and modeling errors. According to the interpolation scheme, the inward normal \mathbf{N}_i of \mathbf{p}_i is

$$\mathbf{N}_i = (\mathbf{n}_i + L_i\Psi) / |\mathbf{n}_i + L_i\Psi|, \quad (15)$$

where $\mathbf{n}_i = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} ((\mathbf{p}_{i'} + \delta'_{i'}) - (\mathbf{p}_{i''} + \delta''_{i''}))$ is the initial normal under gravity with i' and i'' being the indices of \mathbf{p}_i 's two neighbors, and L_i is a constant matrix.

Write $C_1 = (R_3^T, R_4^T)^T$, where R_3 and R_4 are both of dimension 2×4 . Then

$$\mathbf{f}_i = R_i\Psi, \quad (16)$$

Substituting (15) and (16) to (14), we derive the constraints for the set of possible Ψ :

$$\Psi^T R_i^T L_i \Psi + \Psi^T R_i^T \mathbf{n}_i \geq 0, \quad (17)$$

$$(L_i\Psi + \mathbf{n}_i)^T \left(R_i\Psi (R_i\Psi)^T - \frac{(R_i\Psi)^T R_i\Psi}{1 + (\kappa\mu)^2} \right) (L_i\Psi + \mathbf{n}_i) \geq 0, \quad (18)$$

for $i = 3, 4$. In the four constraint equations, all parts other than Ψ are constant. The set of states satisfying the constraints is the configuration space.

C. Goal State

Now we will describe the state that lifts the object doing the least possible work given the configuration. The system under consideration is conservative. Thus, the work done by the fingers is $W = \frac{1}{2}\Psi^T C_1 \Psi$.

Let $J = (f_{1y}, f_{2y})^T$ denote the supporting force vector under only gravity, At the moment of liftoff,

$$F_{sy} = C_2\Psi + J = 0. \quad (19)$$

Since C_2 is fully ranked, the inverse of $C_2 C_2^T$ exists. Thus the solution to equation (19) is $\Psi = -C_2^T (C_2 C_2^T)^{-1} J + \alpha V_1 + \beta V_2$, where $V_1 = (1, 0, 1, 0)^T$ and $V_2 \perp V_1$, obtainable through Gram-Schmidt procedure, are two vectors that span the null space of C_2 , and $\alpha, \beta \in \mathbb{R}$ are projections of Ψ on V_1 and V_2 respectively. Since V_1 is a pure translation, we can choose to set $\alpha = 0$. In that case

$$\Psi = \Psi_c + \beta V_2. \quad (20)$$

where $\Psi_c = -C_2^T (C_2 C_2^T)^{-1} J$. So

$$W = \frac{1}{2}\Psi^T C_1 \Psi^T = a_0 + a_1\beta + a_2\beta^2, \quad (21)$$

where $a_0 = \Psi_c^T C_1 \Psi_c$, $a_1 = 2\Psi_c^T C_1 V_2$, and $a_2 = V_2^T C_1 V_2$. Note that $a_0, a_2 > 0$ given C_1 is positive semi definite and neither V_2 nor Ψ_c lies in C_1 's null space.

Substituting (20) into (17) and (18), we obtain the constraint for the goal state:

$$a_{i1}\beta^2 + a_{i2}\beta + a_{i3} \geq 0 \quad (22)$$

$$a_{i4}\beta^4 + a_{i5}\beta^3 + a_{i6}\beta^2 + a_{i7}\beta + a_{i8} \geq 0 \quad (23)$$

where a_{ij} , $j = 1, \dots, 8$, are constant coefficients that depend only on $L_i, R_i, \mathbf{n}_i, V_2$, and Ψ_c .

Let β^* minimize (21) subject to (22) and (23), Ψ^* given by (20) with $\beta = \beta^*$ is the goal state.

D. Planning

Given the start and goal state and the C-space (reducible to three dimensions using V_1), we use the RRT algorithm [7] to calculate a sequence P of states $\Psi_0, \Psi_1, \dots, \Psi^*$.

The algorithm may generate more states than necessary. Given the output P' from RRT, we sequentially check whether the line segment $\overline{\Psi_{i-1}\Psi_{i+1}}$, $2 \leq i < r-1$, $r = |P'|$ is collision-free. If so, drop Ψ_i . We linearly interpolate the line segment $\Psi_p = t\Psi_a + (1-t)\Psi_b$, where $t \in [0, 1]$. Its substitution into (17) and (18) yields two polynomial constraints on t , that are solvable in constant time. The line is collision free if and only if the solution contains the interval $[0, 1]$. Such algorithm runs in time linear in the path size.

Given any Ψ , the force and normal at each contact point form an angle $\theta_i = \arcsin(\mathbf{f}_i \times \mathbf{N}_i / |\mathbf{f}_i|)$, for $i = 3, 4$. The half friction cone angle is $\theta_\mu = \arctan \mu$. The ratio θ_i / θ_μ indicates how close the force is to the edge of the friction cone, and that how unstable the grasp is at one contact. Therefore the tuple $D = (\theta_1, \theta_2) / \theta_\mu$ indicates the vulnerability of the grasp at Ψ . We use Rejection Sampling algorithm [9] to generate Ψ such that D follows a truncated 2-variate normal distribution with the following probability density function

$$g(D = (x, y)) = \begin{cases} \frac{f(x, e_1, \sigma_1^2) f(y, e_2, \sigma_2^2)}{p}, & \text{if } (x, y) \in [0, 1]^2, \\ 0, & \text{otherwise} \end{cases} \quad (24)$$

where $f(x, e, \sigma^2)$ is the normal probability density at x , taking σ as the standard deviation, and e as the expectation, $i = 1, 2$, and p is the normalization coefficient. Here $\sigma_1 = \sigma_2 = 0.5$ and $e_1 = e_2 = 0$.

IV. GRASP PLANNING ON SOLID OBJECTS

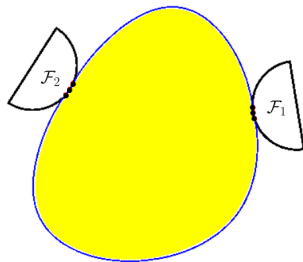


Fig. 2: Grasping solid objects.

We now extend the strategy to solid ones. In Fig. 2, a solid object deforms under gravity. Two rounded fingers are making frictional area contact with the object. As a result of changing contact configuration, the clean formulations in Section III do not hold. Instead, estimations are made to solve the problem.

A. Configuration space

Suppose at the start of grasp, the fingers' positions are $\mathbf{q}_1 = (q_{1x}, q_{1y})^T$ and $\mathbf{q}_2 = (q_{2x}, q_{2y})^T$. The four dimensional state Ψ can always be spanned by a set of four basis vectors $\mathbf{t} = (\sqrt{2}/2, 0, \sqrt{2}/2, 0)^T$, $\mathbf{l} = (0, \sqrt{2}/2, 0, \sqrt{2}/2)^T$, $\mathbf{s} = (q_{2x} - q_{1x}, q_{2y} - q_{1y}, q_{1x} - q_{2x}, q_{1y} - q_{2y})^T / \sqrt{2}[(q_{2x} - q_{1x})^2 + (q_{2y} - q_{1y})^2]$, and $\mathbf{r} = (-q_{1y}, q_{1x}, -q_{2y}, q_{2x})^T / \sqrt{q_{1x}^2 + q_{1y}^2 + q_{2x}^2 + q_{2y}^2}$, which correspond to translation in the x -direction, in the y -direction (lifting), squeezing, and rotation. Again \mathbf{t} is irrelevant to grasping. We do not consider \mathbf{r} either for two reasons: a) the grasp should keep object in roughly the same orientation; and b) the linear elasticity model cannot handle large rotations. Thus our C-space is spanned by \mathbf{l} and \mathbf{s} .

Let $\Phi = (\mathbf{s}, \mathbf{l})^T = (\mathbf{s}, \mathbf{l})^T \Psi$ denote a state. Any Φ in the C-space should constrain the object. We can calculate the contact configuration at both fingers using the event driven algorithm described in [4] for any Φ . Whether the force at every contact region is out of the friction cone indicates whether Φ is out of the C-space.

B. Goal State

At liftoff, the force from the supporting plane becomes 0. Let the set of contacts on the fingers be T and $m = |T|$. Such a state can be interpreted from another perspective: the object is only in contact with the fingers with contact set T , and $\tilde{\mathbf{p}}_{\text{low},y} = 0$, where $\tilde{\mathbf{p}}_{\text{low}}$ is the location of the lowest point of the object.

The contact set stays the same for a certain continuous set of Φ . Within such set,

$$\tilde{\mathbf{p}}_{\text{low},y} = D\bar{\Delta} + b + p_{\text{low},y}, \quad (25)$$

where $\bar{\Delta}$ represents all the contact nodes' displacements, $D = \partial \tilde{\mathbf{p}}_{\text{low},y} / \partial \bar{\Delta}$ is constant, and b is the deformation due to gravity. When sliding is neglected,

$$\bar{\Delta} = A\Phi + B, \quad (26)$$

where A and B are constant matrices. From equation (25) and (26), at liftoff,

$$\Phi = V_0 t - V_1, \quad (27)$$

where $t \in \mathcal{R}$ is a free variable, $V_0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} DA$ and $V_1 = (0, (DB + b + p_{\text{low},y})/b)^T$.

The contact force is linear to the contact displacement

$$\bar{F} = C\bar{\Delta} + F_c, \quad (28)$$

where F_c is some constant due to gravity and compliance to finger shape. Substitute equation (26) and (27) into (28), $\bar{F} = V_2 t + V_3$, where $V_2 = CAV_0$ and $V_3 = CAV_1 + CB + F_c$. To estimate the range of t that prevent the fingers from slipping, we calculate

$F_a = \sum_{i \in T_j} R_i F_i$, where T_j , $j = 1, 2$ is the contact set for finger j . The unit normal $N_i = (N_{ix}, N_{iy})^T$ at point i is rotated to $(0, 1)^T$ by $R_i = \begin{pmatrix} N_{iy} & -N_{ix} \\ N_{ix} & N_{iy} \end{pmatrix}$. The force F_a stays inside the friction cone if $|F_{ax}| \leq \mu |F_{ay}|$ and $F_{ay} \geq 0$. Write $F_a = (f_{a1}t + f_{a2}, f_{a3}t + f_{a4})^T$, then

$$(f_{a1}^2 - f_{a3}^2)t^2 + 2(f_{a1}f_{a2} - f_{a3}f_{a4})t + f_{a2}^2 - f_{a4}^2 \leq 0, \quad (29)$$

$$f_{a3}t + f_{a4} \geq 0, \quad (30)$$

solving which gives us the set of estimated valid t value.

Minimizing the work done by the fingers is equivalent to minimizing the total potential energy at liftoff

$$E = \frac{1}{2} \Delta^T K \Delta + \mathbf{G}^T \Delta, \quad (31)$$

where $\Delta = L\bar{\Delta} + Q$, L and Q are constant matrices that can be derived from equation (1) to (3). Substitute equation (26) and (27) into (31): $E = e_0 + e_1t + e_2t^2$, where e_i , $i = 0, 1, 2$ are obtained through the known constants. The goal state is Φ^* corresponding to t^* that minimizes E .

C. Planning Algorithm

In planning the grasp, we first estimate the contact region by squeezing the object to a degree that it is liftable. Then using this contact region, we estimate the goal state, and perform RRT to find a path. If the contact region is different from initial estimation, we update the estimation with current outcome and iterate the procedure again until the regions stay the same.

V. SIMULATION

In this section, the metric system is used. In Fig. 3(a), an elliptical hollow object sit on the table with two supporting points. Under gravity, it deformed to be Fig. 3(b). Two fingers were then placed where the friction cones are shown (the triangles).

We then run the planning algorithm. A path was planned from $(0, 0, 0, 0)^T$ to the calculated goal $(1.521, 1.162, 1.521, 1.164)^T \times 10^{-2}$. The path planned in the configuration space is mapped to the 2D plane to display. The initial path planned by RRT is shown in Fig. 4(a), where the left (right) subplot corresponds to the trajectory of the left (right) finger, while Fig. 4(b) shows the path after post-process, in which only the third road state in Fig. 4(a) remained.

We executed the planned path in Fig. 4(b). At the starting state, the intermediate state $(-5.21, -1.84, 5.21, -2.23)^T \times 10^{-3}$, and the goal state, the shape of the object is shown in Fig. 3(b), (c), (d) respectively. The pink short lines indicate the contact force at that point.

In Fig. 3(e), the fingers traveled towards the goal state straightly from the starting state. At a state

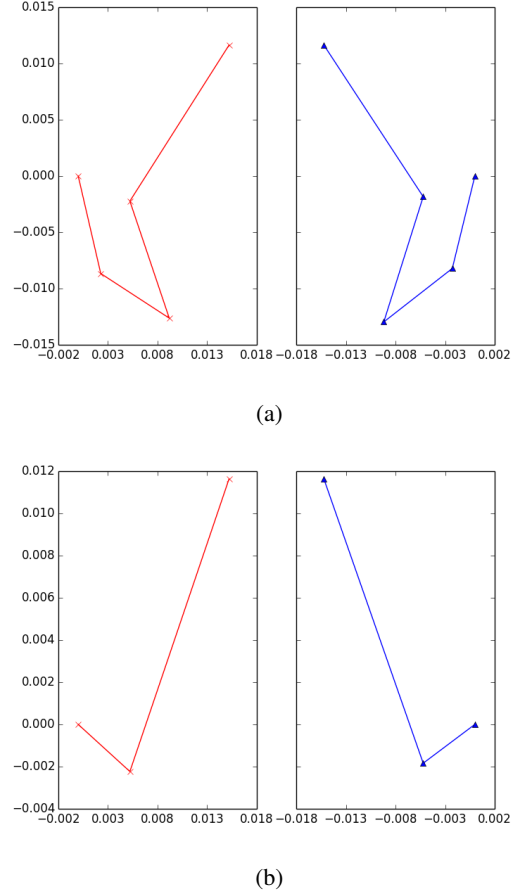


Fig. 4: Planned paths: (a) by RRT, (b) post-processed.

$(4.56, 3.49, 4.56, 3.49)^T \times 10^{-3}$ long the path, the forces were out of the friction cones, resulting in a failed grasp.

Given an object, the planning success rate largely depends on the finger placement. Out of 5 trials, some placements succeeded all 5 times while others 0. We consider the object not graspable at a placement after 5 failures.

VI. DISCUSSION

In this paper, we have introduced a strategy for navigating two fingers to grasp deformable objects. The C-space is described by frictional constraints, and the goal state is calculated to minimize the work done by fingers. The RRT algorithm is used to plan the path.

The work minimization usually results in a goal state that has the grasping force near or on the edge of the friction cone, and thus stability is a trade-off. Reduction of the number of road states may also reduce stability because the new straight line segment may scrape the edge of the C-space. The introduction of safety parameter guarantees a minimum distance between the force and the edge of the friction cone. Using truncated normal

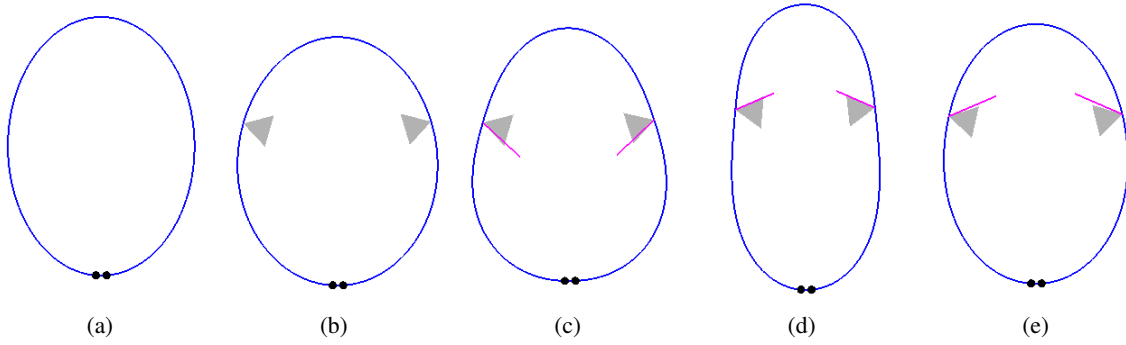


Fig. 3: Grasp planning on an ellipse. Its axes' were 0.164 and 0.118, $(h, w, \rho, E) = (0.0018, 0.012, 1000, 5 \times 10^7)$.

distribution reduces the chance of having intermediate states near the edge. However, both improvements are at the price of reducing the rate of successful planning.

Future work will include extending to 3D objects and experiment validations. The extension can be made at the price of more complicated formulation and calculation. Estimations can be made similarly as Section IV. In application, the needed physical parameters can all be obtained or measured, and the shape of the object can be obtained by scanners or 3D cameras.

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APPENDIX I

PROOF OF LEMMA 2

Proof: Let $\bar{\Delta} = (X^T, 0)^T$, where $X \in \mathbb{R}^4$ is an arbitrary vector. That $E = \frac{1}{2}X^T C_1 X \geq 0$ indicates C_1 is positive semi-definite.

When $X = (1, 0, 1, 0)^T$, $\bar{\Delta}$ lies in the null space of C . So $E = \frac{1}{2}X^T C_1 X = \frac{1}{2}\bar{\Delta}^T C \bar{\Delta} = 0$. Therefore $(1, 0, 1, 0)^T$ lies in C_1 's null space and $\text{rank}(C_1) \leq 3$.

Suppose for contradiction that $\text{rank}(C_1) < 3$. Then $\exists X' \perp (1, 0, 1, 0)^T$, $|X'| \neq 0$, such that $C_1 X' = C \bar{\Delta}' = 0$, where $\bar{\Delta}' = (X'^T, 0)^T$. Thus $\bar{\Delta}'$ must be a linear combination of C 's the other two null vectors:

$$(X'^T, 0)^T = x_1(0, 1, 0, 1, 1, 1)^T + x_2(-p_{3y}, p_{3x}, -p_{4y}, p_{4x}, p_{1x}, p_{2x})^T,$$

where $x_1, x_2 \in \mathbb{R}$ and $x_1^2 + x_2^2 \neq 0$. Take the last two equations out from the above equation

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & p_{1x} \\ 1 & p_{2x} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Given $p_{1x} \neq p_{2x}$, $x_1 = x_2 = 0$, which is a contradiction. Thus $\text{rank}(C_1) = 3$. ■