1. Formulate the problem of scheduling independent machines as an integer programming problem. Give the linear programming relaxation and the dual of the relaxation.

2. In the minimum knapsack problem, we are given a set of items \( \{1, \ldots, n\} \), where item \( i \) has a positive integer cost \( c_i \) and a positive integer size \( s_i \), and a positive integer \( B \). The goal is to find \( S \subseteq \{1, \ldots, n\} \) such that \( \sum_{i \in S} c_i \) is minimized, subject to the constraint that \( \sum_{i \in S} s_i \geq B \). Give a FPTAS for this problem.

3. Consider the following variant of the problem of scheduling independent tasks where machine \( j, j \in \{1, \ldots, m\} \) has speed \( s_j \in \mathbb{Q}^+ \). Thus, the time to execute job \( i \) on machine \( j \) is \( \frac{p_i}{s_j} \). Give a PTAS for this problem for the case where \( m = 2 \).

4. Here is an alternative approximation algorithm for scheduling independent tasks:

   Let \( k \) be some specified fixed integer. First, find an optimum schedule for the \( k \) longest jobs. Then, schedule the remaining jobs using the LPT rule.

   (a) Show that this algorithm returns a solution with makespan that is at most
   \[
   1 + \frac{1 - 1/m}{1 + \lfloor k/m \rfloor}
   \]
   times the optimum.

   (b) Explain how to use this approach to obtain a PTAS for fixed \( m \). Why isn’t this a PTAS when \( m \) is not fixed?

5. Show that given any instance \( I \) of the bin packing problem, First-Fit Decreasing (FFD) returns a solution that satisfies:

   \[
   FFD(I) \leq 1.5OPT(I) + 1.
   \]
(Hint: Partition the set of items \( \{1, \ldots, n\} \) by size as follows:

\[
\begin{align*}
A &= \{i : s_i > 2/3\} \\
B &= \{i : 2/3 \geq s_i > 1/2\} \\
C &= \{i : 1/2 \geq s_i > 1/3\} \\
D &= \{i : 1/3 \geq s_i\}
\end{align*}
\]

6. (This problem can be solved in place of one of the previous four.) Conduct an experimental study of heuristics for either bin packing or multiprocessor scheduling. Evaluate different heuristics from the point of view of performance ration and running time. You must clearly explain your experimental setup and give an understandable explanation of your experimental results.