Assignment 2

Comm S 477/577

Due on Sep 11, 2003

You should turn in a hard copy of your code and sample runs on specified inputs. Store the coefficients of a polynomial \( p(x) \) of degree \( n \) in an array, say \( a[] \), with \( a[0] \) being the constant term and \( a[n] \) the leading coefficient.

1. (14 pts) Polynomial reciprocals.

(a) (8 pts) Write a procedure \texttt{Reciprocal}(n, a[], s[]) which takes as input the coefficient array \( a[] \) of a polynomial \( p(x) \) of degree \( n \), where \( n = 2^k - 1 \). It outputs an array \( s[] \) that stores the coefficients of the reciprocal \( s(x) \) such that \( p(x)s(x) = x^{2m} + t(x) \) where \( t(x) \) is a polynomial of degree less than \( n \). Polynomial multiplications must be carried out in time \( O(n \lg n) \).

(b) (6 pts) Use the procedure \texttt{Reciprocal} to find the reciprocals of the following polynomials:

\[
\begin{align*}
p_1(x) &= x^7 - x^6 + x^5 - 3x^4 + x^3 - x^2 + 2x + 1, \\
p_2(x) &= 10x^{15} + x^{12} - 3x^{10} - 4x^9 + 11x^8 + 15x^7 + 12x^6 - 14x^5 - 24x^4 + 23x^3 - 51x^2 - 64.
\end{align*}
\]

2. (20 pts) Polynomial division. Suppose you are given two polynomials

\[
\begin{align*}
u(x) &= u_mx^m + \cdots + u_1x + u_0, \\
v(x) &= v_nx^n + \cdots + v_1x + v_0,
\end{align*}
\]

where \( m \geq n \), \( u_m \neq 0 \), and \( v_n \neq 0 \).

(a) (6 pts) Describe in pseudocode a fast algorithm that computes the quotient \([u(x)/v(x)]\). You may name the parameters \( u(x) \) and \( v(x) \). Analyze the running time of your algorithm.

(b) (10 pts) Implement your algorithm as a procedure \texttt{polyDivide}(m, n, u[], v[], q[], r[]), where \( q[] \) and \( r[] \) store the coefficients of the quotient and remainder polynomials, respectively.

(c) (4 pts) Use \texttt{polyDivide} to compute the quotient and remainder when \( u(x) = p_2(x) \) and \( v(x) = p_1(x) \) in Problem 1(b) and when

\[
\begin{align*}
u(x) &= -22.4x^9 + 1.13x^8 - 24x^7 - 5.167x^6 + 14x^5 - 200x^4 - x^3 + 451x^2 - 70.3x + 101, \\
v(x) &= 8.31x^5 - 50x^4 + 0.23x^2 - 111x - 23.
\end{align*}
\]

3. (16 pts) Greatest common divisors.

(a) (6 pts) Show that Euclid’s algorithm for polynomials can be extended to find polynomials \( a(x) \) and \( b(x) \) such that

\[
a(x)u(x) + b(x)v(x) = \gcd(u(x), v(x)).
\]
What are the degrees of the polynomials $a(x)$ and $b(x)$ that are computed by this extended algorithms?

(b) (8 pts) Write a procedure $\text{Extended-Euclid-Poly}(m, n, u[], v[], k, l, a[], b[])$ that computes the polynomials $a(x)$ and $b(x)$ as defined in part (a) and store them in the arrays $a[]$ and $b[]$. Here $k$ and $l$ store the degrees of $a(x)$ and $b(x)$. You do not have to implement the fast gcd algorithm that employs the subroutine HGCD.

(c) (2 pts) Find $a(x)$ and $b(x)$ when $u(x) = x^{21} - 1$ and $v(x) = x^{13} - 1$. 