19. Show that each of the following problems are in NP by giving a non-deterministic algorithm.

• Let $G = (V, E)$ be an undirected graph and $D \subseteq V$. We say that $D$ is a d-set if for every vertex $u$ that is not in $D$ there exists a vertex $v \in D$ such that $(u, v) \in E$.

$$L = \{ \langle G, k \rangle \mid G \text{ has a d-set of size } \leq k \}.$$

• An undirected bi-partite graph $G = (L, R, E)$ is called a $d$-graph if the following holds:

For every $S \subseteq L$ with $|S| \geq \sqrt{|L|}$, the cardinality of $N(S)$ is at least $\frac{3}{4}|R|$. Here $N(S) = \{ u \in R \mid \text{there exists } v \in S \text{ such that } (u, v) \in E \}$.

$$L = \{ G \mid G \text{ is not a d-graph} \}$$

20. Show that the above languages are in NP by showing that they are in PV.

21. A function $f : \Sigma^* \rightarrow \Sigma^*$ is a permutation if $f$ is a bijection and $|x| = |f(x)|$ for every $x$. Let $f$ be a polynomial-time computable permutation. Consider the following language:

$$L = \{ \langle y, i \rangle \mid \text{ith bit of } f^{-1}(y) \text{ is 1} \}.$$

(a) Show that $L$ is in $NP$.
(b) Show that $\overline{L}$ is in $NP$.
(c) Show that $L$ is in $E$.

22. Show that if $L$ is in $PV$, then $L^*$ is in $PV$.

Guidelines.

• You are allowed to discuss with your classmates, while doing your homework. However, I strongly suggest that you think about the problems on your own before discussing.

• Definition of classmates: Students who are taking CS 531 in Spring 14.

• However, You should write the final solutions alone, without consulting your classmates. Your writing should demonstrate that you understand the proofs completely. If I suspect that you wrote the proofs without understanding, I may ask you to explain the proofs to me in person. In such scenarios, failure to explain proofs will be taken as evidence of academic dishonesty.
• For each problem, you should acknowledge the students with whom you discussed. This will not affect your grade. Failure to acknowledge is considered *academic dishonesty*, and it will affect your grade.

• Any student found guilty of academic dishonesty will receive “F” in the course.

• When proofs are required, make them both clear and rigorous. Do not hand wave. Even when proofs are not required, you should justify your answers and explain your work.

• Typewritten homework is appreciated, and neat presentation is required. In order to give you credit, the TA must be able to read what you have written.

• Late home works are not accepted.