Submission Instructions
This assignment must be submitted by 5 PM of its due date at the CS 511 drop box.

Reading Assignment
Chapter 8 of K & T.

Problem Set

1. (10 points) Suppose you have a procedure $P$ which given a graph $G$ and an integer $K$ runs in polynomial time and returns “yes” if $G$ has an independent set of size $K$ and returns “no” otherwise. Show that you can use $P$ to develop a polynomial-time algorithm that returns an independent set of size $K$ in $G$, if one exists.

2. (10 points) For each of the problems below, prove that it is NP-complete by showing that it is a generalization of some NP-complete problem.

   (a) (3 points) SUBGRAPH ISOMORPHISM: Given input two undirected graphs $G$ and $H$, determine whether or not $G$ is a subgraph of $H$. That is, determine whether by deleting certain vertices and edges of $H$ we obtain a graph that is, up to renaming of vertices, identical to $G$.

   (b) (3 points) SPARSE SUBGRAPH: Given a graph $G$ and two integers $a$ and $b$, does $G$ have a set of $a$ vertices such that there are at most $b$ edges between them.

   (c) (4 points) RELIABLE NETWORK: We are given two $n \times n$ matrices, a distance matrix $d_{ij}$ and a connectivity requirement matrix $r_{ij}$, as well as a budget $b$. Does there exist a graph $G = (V = \{1, 2, \ldots, n\}, E)$ such that (i) the total cost
of all edges is at most $b$ and (ii) between any two distinct vertices $i$ and $j$ there are $r_{ij}$ vertex-disjoint paths. \textit{(Hint: Suppose that all $d_{ij}$’s are 1 or 2, $b = n$, and all $r_{ij}$’s are 2. Which well known NP-complete problem is this?)}

3. (10 points) Exercise 2, page 505.


5. (10 points) Exercise 7, page 507.

6. (10 points) Exercise 9, page 508.

\textbf{Note.} We reserve the right to grade only a subset of the problems assigned. Which problems will be graded will be decided after the submission deadline.