1.2 Proposer algorithm

In the Phase 1, proposer $p$ sends a $p1$ message (looks like a typo in the paper?) that is telling acceptors not to accept any other proposal will a ballot number $\leq \text{ballot}_p$ (since it contains a ballot number). If an acceptor already replied to $p1$ message that contained a higher ballot number, then it sends a $p\text{abort}$ message back to $p$. Otherwise, it informs the proposer of a value previously accepted, and the corresponding ballot number using a $p\text{ack}$ message.

The main point is when a proposer receives one of those messages ($p\text{ack}$ or $p\text{abort}$) from a quorum of acceptors:

- if any acceptor replies with $p\text{abort}$, the proposer returns abort;
- otherwise, if all messages from a quorum of acceptors are $p\text{ack}$ messages, then it does the $\text{leaderSelect}$ part of the protocol, i.e. adopts the proposal value with the highest ballot number in the received quorum.

Remark 2 This is what guarantees the requirement that if some learner learns the value $v$ with a ballot number $b$, the acceptors can only accept $v$ in later ballots higher than $b$.

In phase 2, proposer $p$ sends its proposal value $v$ to all the acceptors with the same old ballot number $\text{ballot}_p$ in a $p2$ message. Unless some other proposer started in Phase 1 with a higher ballot number in the meantime, a quorum of acceptors is going to accept value $v$. Otherwise, if some proposer did start Phase 1 with a higher ballot number, then an acceptor may send a $p\text{2abort}$ message to the proposer, causing the proposer to return abort.

On accepting $v$, acceptor $a$ stores $v$ locally and sends a $p2\text{ack}$ message containing ballot number $\text{ballot}_p$ and value $v$ to all learners. A learner learns a value if it receives $p2\text{ack}$ messages from a quorum of acceptors with the same value and same ballot number.

Showing that SynodOFC maintains consensus Validity is straightforward. OFC-termination can be proved by assuming a single proposer keeping proposing to SynodOFC with ballot numbers that keep increasing. In the next part, we focus on proving Agreement.
1.3 Agreement

Definition 3 If some learner learns a value $v$ by receiving a quorum of $(p2ack, b, v)$ messages, we say that learner learns $v$ in $b$. If a quorum of acceptors accepts $v$ by receiving $(p2, b, v)$ messages, we say $v$ is chosen in $b$.

Lemma 3 If a learner learns $v$ in ballot $b$, then $v$ is chosen in $b$.

Proof: If learner receives $(p2ack, b, v)$ from a quorum, the same quorum received $(p2, b, v)$ messages in Phase 2, so by definition, $v$ is chosen in $b$.

Lemma 4 If a value $v$ is chosen in $b$, then an acceptor can only accept $v$ in higher ballots.